CODE - A

TEST ID 001915

NTPL

ALL INDIA TEST SERIES

Batch - 1907+1908

JEE (Main) - 2019

PART TEST - 5

Time : 3 Hours ***Maximum Marks : 360***

Syllabus Covered

Physics	:	Electrostatics.
Chemistry	:	Ionic Equilibrium, Electrochemistry, Hydrocarbon.
Mathematics	:	Area, Differential Equation, Straight Lines.

Please read the instructions carefully. You are allotted 5 minutes specifically for this purpose.

You are not allowed to leave the Examination Hall before the end of the test.

INSTRUCTIONS

A. General :

1. This booklet is your Question Paper containing **90 questions**.
2. The Question Paper **CODE & TEST ID** is printed on the right hand top corner of this booklet. This should be entered on the OMR Sheet.
3. Fill the bubbles completely and properly using a **Blue/Black Ball Point Pen** only.
4. No additional sheets will be provided for rough work.
5. Blank papers, clipboards, log tables, slide rules, calculators, cellular phones, pagers, and electronic gadgets in any form are not allowed to be carried inside the examination hall.
6. The answer sheet, a machine-readable Optical mark recognition sheet (OMR Sheet), is provided separately.
7. DO NOT TAMPER WITH / MUTILATE THE OMR OR THE BOOKLET.
8. Do not break the seals of the question-paper booklet before being instructed to do so by the invigilator.

B. Question paper format & Marking Scheme :

9. The question paper consists of **3 parts** (Physics, Chemistry and Mathematics).
10. The test is of **3 hours** duration. Each question has 4 choices (A), (B), (C) and (D), out of which **ONLY ONE** is correct. Each question carries **+4 marks** for correct answer and **-1 mark** for wrong answer.

Name of the Candidate (in Capitals) ____________________

Test Centre ____________________ Centre Code ____________________

Candidate's Signature ____________________ Invigilator's Signature ____________________

PHYSICS

SECTION – I

This section contains ***30 questions.*** Each question has 4 choices (A), (B), (C) and (D) for its answer, out of which ***ONLY ONE*** is correct. Each question carries **+4 marks** for correct answer and **–1 mark** for wrong answer.

1. Two spheres of radii r and R carry charges q and Q respectively. When they are connected by a wire, there will be no loss of energy of the system if

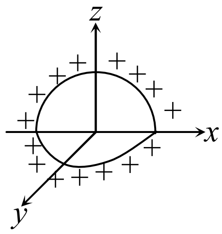
(a) $qr = QR$ (b) $qR = Qr$ (c) $qr^2 = QR^2$ (d) $qR^2 = Qr^2$

2. The electric potential V at any point P (x, y, z) in space is given by $V = 4x^2$ (where V is in volt and x in meter). The electric field at the point (1m, 2m) in volt/metre is

(a) 8 along negative x-axis (b) 8 along positive x-axis

(c) 16 along negative x-axis (d) 16 along positive z-axis

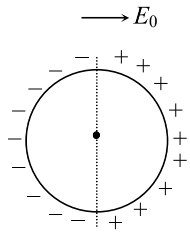
3. In the given figure two semicircular wire is connected which are in x-y and x-z plane respectively. And $+2q_0$ charge is distributed over it, then what will be the magnitude of electric field intensity at the origin.



(a) $\dfrac{q_0}{\pi\varepsilon_0 R^2}$ (b) $\dfrac{4q_0^2}{\pi\varepsilon_0 R^2}$

(c) $\dfrac{q_0}{\sqrt{2}\pi^2\varepsilon_0 R^2}$ (d) $\dfrac{q_0}{\sqrt{2}\pi\varepsilon_0 R^2}$

4. Half part of ring is uniformly positively charged and other half is uniformly negatively charged. Ring is in equilibrium in uniform electric field as shown and free to rotate about an axis passing through its centre and perpendicular to plane. The equilibrium is



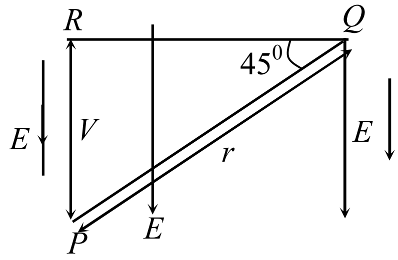
(a) stable

(b) unstable

(c) neutral

(d) can be stable or unstable

5. A unit positive charge moves in an electric field E along the path PQR. The potential difference between points R and P will be



(a) zero (b) $E.\,r$

(c) $\frac{E.r}{\sqrt{2}}$ (d) $E.r\sqrt{2}$

6. A and B are similarly charged bodies which repel with force F. Another uncharged sphere C of same size and material is touched with B and removed. It is kept at mid-point of distance between A and B. Force on C is

(a) $\frac{F}{2}$ (b) $2F$ (c) F (d) $\frac{2}{3}F$

7. 8 drops of equal radius coalesce to form a bigger drop. By what factor the charge and potential change?

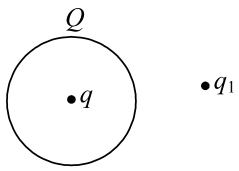
(a) 8, 4 (b) 8, 8 (c) 6, 8 (d) 8, 10

8. Two thin wire rings each having a radius R are placed at a distance d apart with their axis coinciding. The charges on the two rings are $+Q$ and $-Q$. The potential difference between the centres of the two rings is

(a) zero (b) $\frac{Q}{4\pi\varepsilon_0}\left[\frac{1}{R}-\frac{1}{\sqrt{R^2+d^2}}\right]$

(c) $\frac{QR}{4\pi\varepsilon_0 d^2}$ (d) $\frac{Q}{2\pi\varepsilon_0}\left[\frac{1}{R}-\frac{1}{\sqrt{R^2+d^2}}\right]$

9. A thin, metallic spherical shell contains a charge Q on it. A point charge q is placed at the centre of the shell another charge q_1 is placed out side as shown. All three charges are positive. The electrostatic force on the charge at the centre is



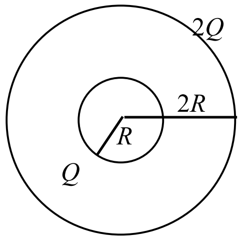
(a) towards left (b) towards right

(c) upward (d) zero

10. A solid conducting sphere having a charge Q is surrounded by an uncharged concentric conducting hollow spherical shell. The potential difference between the surface of solid sphere and the outer surface of the hollow shell is V. If the shell is now given a charge of $-3Q$, the new potential difference between the same two surfaces is

(a) V (b) $2\ V$ (c) $4\ V$ (d) $-2\ V$

11. Two concentric hollow spherical shell of radius R and $2R$ having charges Q and $2Q$ as shown. Find the total electric potential energy of arrangement.



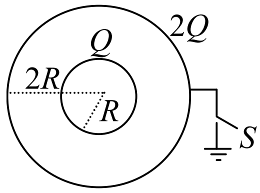
(a) $\frac{5}{2}\frac{KQ^2}{R}$ (b) $\frac{KQ^2}{R}$

(c) $\frac{3KQ^2}{R}$ (d) $\frac{2KQ^2}{R}$

12. Two point charges $+4e$ and e are kept at distance x apart. At what distance a charge q must be placed from charge $+e$, so that q is in equilibrium

(a) $\frac{x}{2}$ (b) $\frac{2x}{3}$ (c) $\frac{x}{3}$ (d) $\frac{x}{6}$

13. Two concentric spherical shell of radius R and $2R$ having initial charges Q and $2Q$ respectively as shown. On closing the switch S charge flow from outer sphere to earth is



(a) Q

(b) $-Q$

(c) $-3Q$

(d) $3Q$

14. Two equal point charges are fixed at $x = -a$ and $x = +a$ on the x-axis. Another point charge Q is placed at the origin. The change in the electrical potential energy of system, when it is displaced by a small distance x along the x-axis, is approximately proportional to

(a) x (b) x^2 (c) x^3 (d) $1/x$

15. Two identical thin rings, each of radius R are coaxially placed at a distance R apart. If Q_1 and Q_2 are the charges uniformly spread on the two rings, the work done by the electric field in moving a charge q from the centre of first ring to the centre of the second ring is

(a) zero

(b) $\frac{q}{4\pi\varepsilon_0\sqrt{2}R}(Q_1 - Q_2)(\sqrt{2}-1)$

(c) $\frac{q\sqrt{2}}{4\pi\varepsilon_0 R}(Q_1+Q_2)$

(d) $\frac{(\sqrt{2}+1)q(Q_1+Q_2)}{\sqrt{2}4\pi\varepsilon_0 R}$

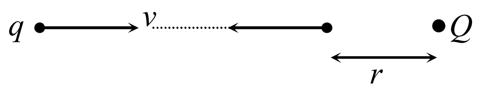
16. The maximum electric field intensity on the axis of a uniformly charged ring of charge q and radius R will be

(a) $\frac{1}{4\pi\varepsilon_0}\frac{q}{3\sqrt{3}R^2}$ (b) $\frac{1}{4\pi\varepsilon_0}\frac{2q}{3R^2}$ (c) $\frac{1}{4\pi\varepsilon_0}\frac{2q}{3\sqrt{3}R^2}$ (d) $\frac{1}{4\pi\varepsilon_0}\frac{2q}{2\sqrt{2}R^2}$

17. An electric field is given by $\vec{E} = \left(y\hat{i} + x\hat{j}\right)$ N/C . The work done by electric field in moving a 1C charge from position vector $\vec{r}_A = \left(2\hat{i} + 2\hat{j}\right)$m to $\vec{r}_B = \left(4\hat{i} + \hat{j}\right)$m is

(a) + 4 J (b) – 4 J (c) + 8 J (d) zero

18. A particle carrying a charge q is moving with a speed v towards a fixed particle carrying a charge Q at large distance. It approaches Q up to a certain distance r and then returns as shown in the figure. If charge q were moving with a speed $2v$ at large distance, the distance of the closest approach would be



(a) r (b) $2r$ (c) $\frac{r}{2}$ (d) $\frac{r}{4}$

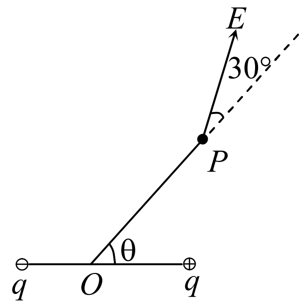
19. n identical charge particle are placed on the vertices of a regular polygon of n sides of side length a. One of the charge particle is released from polygon. When this particle reaches a far of distance, another particle adjacent to the first particle is released. The difference of kinetic energies of both the particles at infinity is k. Magnitude of charge is

(a) $\sqrt{4\pi\varepsilon_0 ak}$ (b) $\frac{k}{4\pi\varepsilon_0 a}$ (c) $\frac{k}{a}$ (d) $\sqrt{ka}$

20. An electric field given by $\vec{E} = 4\hat{i} - 3(y^2 + 2)\hat{j}$ pierces Gaussian cube of side 1m placed at origin such that its three sides represents x, y and z axes. The net charge enclosed within the cube is

(a) $4\varepsilon_0$ (b) $3\varepsilon_0$ (c) $5\varepsilon_0$ (d) zero

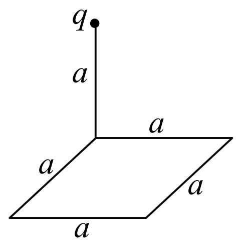
21. If net electric field E due to dipole at point P makes an angle 30° with the line OP as shown, then find the value of angle θ.



(a) $\tan^{-1}\left(\frac{1}{\sqrt{2}}\right)$ (b) $\tan^{-1}\left(\sqrt{2}\right)$

(c) $\tan^{-1}\left(\frac{2}{\sqrt{3}}\right)$ (d) $\tan^{-1}\left(\frac{\sqrt{3}}{2}\right)$

22. A point charge q is placed at a height a from vertex of square of side a as shown. The electric flux through the square is



(a) $\frac{q}{\varepsilon_0}$ (b) zero

(c) $\frac{q}{6\varepsilon_0}$ (d) $\frac{q}{24\varepsilon_0}$

23. The electric potential at a point (x, y) is given by: $V = -Kxy$. The electric field intensity at a distance r from the origin varies as

(a) r^2 (b) r (c) $2r$ (d) $2r^2$

24. The magnitude of electric field intensity at point B (2, 0, 0) due to a dipole of dipole moment, $\vec{P} = \hat{i} + \sqrt{3}\hat{j}$ kept at origin is (assume that the point B is at large distance from the dipole and $k = \frac{1}{4\pi\varepsilon_0}$)

(a) $\frac{\sqrt{13}k}{8}$ (b) $\frac{\sqrt{13}k}{4}$ (c) $\frac{\sqrt{7}k}{8}$ (d) $\frac{\sqrt{7}k}{4}$

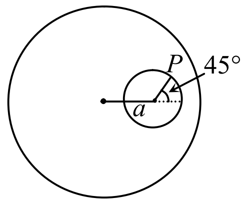
25. For spherical charge distribution given as

$$\begin{cases} \rho = \rho_0\left(1-\dfrac{r}{3}\right) , & \text{when } r \le 3 \\ \rho = 0 , & \text{when } r > 3 \end{cases}$$

(where r is the distance from the centre of spherical charge distribution). The electric field intensity is maximum for the value of

(a) $r = 1.5$ (b) $r = 2$ (c) $r = 4.5$ (d) at infinity

26. A cavity of radius r is made inside a solid sphere. The volume charge density of the remaining sphere is ρ. An electron (charge e, mass m) is released inside the cavity from point P as shown in figure. The centre of sphere and centre of cavity are separated by a distance a. The time after which the electron again touches the sphere is

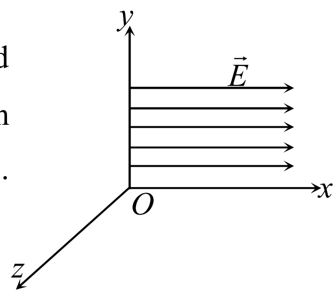


(a) $\sqrt{\dfrac{6\sqrt{2}r\varepsilon_0 m}{e\rho a}}$ (b) $\sqrt{\dfrac{\sqrt{2}r\varepsilon_0 m}{e\rho a}}$ (c) $\sqrt{\dfrac{6r\varepsilon_0 m}{e\rho a}}$ (d) $\sqrt{\dfrac{r\varepsilon_0 m}{e\rho a}}$

27. An electric dipole is placed along the x-axis at the origin O. A point P is at a distance of 20cm from the origin such that OP makes an angle $\dfrac{\pi}{3}$ with the x-axis. If the electric field at P makes an angle θ with the positive direction of x-axis, the value of θ would be

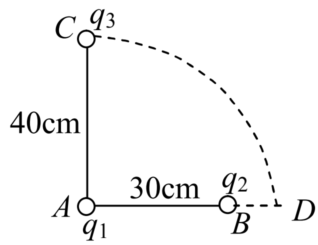
(a) $\dfrac{\pi}{3}$ (b) $\dfrac{\pi}{3}+\tan^{-1}\left(\dfrac{\sqrt{3}}{2}\right)$ (c) $\dfrac{2\pi}{3}$ (d) $\tan^{-1}\left(\dfrac{\sqrt{3}}{2}\right)$

28. There is uniform electric field $\vec{E} = E_0\hat{i}$ as shown in figure. Two charged particles q_1 and q_2 and masses m_1 and m_2 are projected from point O with velocities $\vec{v}_1$ and $\vec{v}_2$ at $t = 0$. At $t = 2t_0$ their velocities become $\vec{v}_1{}'$ and $\vec{v}_2{}'$. Then find $|(m_1\vec{v}_1' + m_2\vec{v}_2') - (m_1\vec{v}_1 + m_2\vec{v}_2)|$ (gravity is absent)



(a) $(q_1+q_2)\, E_0\, t_0$ (b) $2(q_1+q_2)\, E_0\, t_0$ (c) $\dfrac{1}{2}(q_1+q_2)\, E_0\, t_0$ (d) zero

29. Two charges q_1 and q_2 are placed 30 cm apart as shown. A third charge q_3 is moved along the arc of a circle of radius 40 cm from *C* to *D*. The change in the potential energy of the system is $\frac{q_3}{4\pi\varepsilon_0}k$, where *k* is



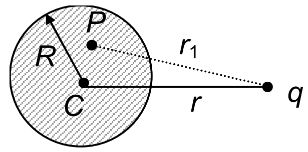
(a) $8q_2$

(b) $8q_1$

(c) $6q_2$

(d) $6q_1$

30. A point charge is placed at a distance *r* from center of a conducting neutral sphere of radius *R* ($r > R$). The potential at any point *P* inside the sphere at a distance r_1 from point charge due to induced charge of the sphere is given by [$k = \frac{1}{4\pi\varepsilon_0}$]



(a) kq/r_1 (b) kq/r

(c) $kq/r - kq/r_1$ (d) $-kq/R$

CHEMISTRY

SECTION – II

This section contains ***30 questions.*** Each question has 4 choices (A), (B), (C) and (D) for its answer, out of which ***ONLY ONE*** is correct. Each question carries **+4 marks** for correct answer and **–1 mark** for wrong answer.

31. The solubility of a sparingly soluble salt AB_2 in water is 1.0×10^{-5} mol litre^{-1}. Its solubility product is :

(a) 10^{-15} (b) 10^{-10} (c) 4×10^{-15} (d) 4×10^{-10}

32. A buffer solution contains 1 mole of $(NH_4)_2 SO_4$ and 1 Mole of NH_4OH ($K_b = 10^{-5}$). The pH of solution will be :

(a) 5 (b) 9 (c) 5.3010 (d) 8.6990

33. pH of water is 7.0 at 25ºC. If water is heated to 70ºC, the :

(a) pH will decrease and solution becomes acidic

(b) pH will increase

(c) pH will remain constant as 7

(d) pH will decrease but solution will be neutral

34. For, $H_3PO_4 + H_2O \rightleftharpoons H_3O^+ + H_2PO_4^-$; K_{a_1}

$H_2PO_4^- + H_2O \rightleftharpoons H_3O^+ + HPO_4^{2-}$; K_{a_2}

$HPO_4^{2-} + H_2O \rightleftharpoons H_3O^+ + PO_4^{3-}$; K_{a_3}

The correct order of K_a values is :

(a) $K_{a_1} > K_{a_2} < K_{a_3}$ (b) $K_{a_1} < K_{a_2} < K_{a_3}$ (c) $K_{a_1} > K_{a_2} > K_{a_3}$ (d) $K_{a_1} < K_{a_2} > K_{a_3}$

35. The degree of hydrolysis of a salt of weak acid and weak base in its 0.1 M solution is found to be 50%. If the molarity of the solution is 0.2 M, the percentage hydrolysis of the salt should be :

(a) 100% (b) 50% (c) 25% (d) None of these

36. pH of water is 7. When a substance Y is dissolved in water, the pH becomes 13. The substance Y is a salt of –

(a) weak acid and weak base
(b) strong acid and strong base
(c) strong acid and weak base
(d) weak acid and strong base

37. The rate constant at 25°C for the reaction of NH_4^+ & OH^- to form NH_4OH is 4×10^{10} M^{-1} sec^{-1} & ionisation constant of aq. NH_3 is 1.8×10^{-5}. The rate constant of proton transfer to NH_3 is –

(a) 1.8×10^{-5}
(b) $7.2 \times 10^{+5}$
(c) 3.6×10^{5}
(d) 4.2×10^{-5}

38. If $E^{\circ}_{Fe^{2+}/Fe}$ is x_1, $E^{\circ}_{Fe^{3+}/Fe}$ is x_2 then $E^{0}_{Fe^{3+}/Fe^{2+}}$ will be :

(a) $3x_2 - 2x_1$
(b) $x_2 - x_1$
(c) $x_2 + x_1$
(d) $2x_1 + 3x_2$

39. The EMF of the cell Ni | Ni^{2+} || Cu^{2+} | Cu(s) is 0.59 volt. The standard reduction electrode potential of copper electrode is 0.34 volt. The standard reduction electrode potential of nickel electrode will be

(a) 0.25 volt
(b) – 0.25 volt
(c) – 0.50 volt
(d) – 0.025 volt

40. The standard oxidation potentials of Zn and Ag is water at 25ºC are,

$Zn(s) \longrightarrow Zn^{2+} + 2e$

$[E^{o} = 0.76\ V]$

$Ag(s) \longrightarrow Ag^{+} + e$

$E^{o} = -0.80\ V$

Which reaction actually takes place :

(a) $Zn(s) + 2Ag^{+}(aq) \longrightarrow Zn^{2+} + 2Ag(s)$
(b) $Zn^{2+} + 2Ag^{+}(s) \longrightarrow 2Ag^{+}(aq) + Zn(s)$
(c) $Zn(s) + 2Ag(s) \longrightarrow Zn^{2+}(aq) + Ag^{+}(s)$
(d) $Zn^{2+}(aq) + Ag^{+}(aq) \longrightarrow Zn(s) + Ag(s)$

41. The reagent used for the conversion

$CH_3CH_2COOH \longrightarrow CH_3CH_2CH_3$, is

(a) $LiAlH_4$
(b) sodalime
(c) red P and concentrated HI
(d) amalgamated zinc and concentrated HCl

42. Pure methane can be produced by

(a) Wurtz reaction
(b) Kolbe's electrolytic method
(c) sodalime decarboxylation
(d) reduction with H_2

43. Arrange the following in the decreasing order of their boiling points.

A. *n*-butane
B. 2-methylbutane
C. *n*-pentane
D. 2, 2-dimethylpropane

(a) A > B > C > D
(b) B > C > D > A
(c) D > C > B > A
(d) C > B > D > A

44. In a mixture of *n*-hexadecane and alpha-methyl naphthalene, the percentage of the later is 10. The value of cetane number is

(a) 110
(b) 90
(c) 10
(d) 0

45. 2-phenylpropene on acidic hydration gives

(a) 2-phenyl-2-propanol
(b) 2-phenyl-1-propanol
(c) 3-phenyl-l-propanol
(d) 1-phenyl-2-propanol

46. *n*-propyl bromide on treatment with ethanolic potassium hydroxide produces

(a) propanol-1
(b) propene
(c) propanol-2
(d) ethyl propyl ether

47. A gas decolourises alkaline $KMnO_4$ solution but does not give precipitate with $AgNO_3$. It is

(a) CH_4
(b) C_2H_4
(c) C_2H_2
(d) C_2H_6

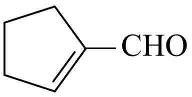
48. Ozonolysis of 2,3-dimethyl- 1-butene followed by reduction with zinc and water gives

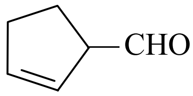
(a) methanoic acid and 3-methyl-2-butanone
(b) methanal and 2-methyl-2-butanone
(c) methanal and 3-methyl-2-butanone
(d) methanoic acid and 2-methyl-2-butanone

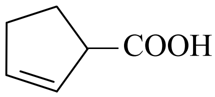
49. Which of the following is the predominant product in the reaction of HOBr with propene?

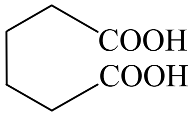
(a) 2-bromo-1 -propanol
(b) 3-bromo-1 -propanol
(c) 2-bromo-2-propanol
(d) 1-bromo-2-propanol

50. Cyclohexene on ozonolysis followed by reaction with zinc dust and water gives compound *E*. Compound *E* on further treatment with aqueous KOH yields compound *F*. Compound *F* is

(a) 

(b) 

(c) 

(d) 

51. $CH_3-CH=CH_2+NOCl \longrightarrow P$. Identify the product P.

(a) $CH_3-\underset{Cl}{\underset{|}{CH}}-\underset{NO}{\underset{|}{CH_2}}$ (b) $CH_3-\underset{NO}{\underset{|}{CH}}-\underset{Cl}{\underset{|}{CH_2}}$ (c) $CH_3CH_2-\overset{NO}{\overset{|}{\underset{Cl}{\underset{|}{CH}}}}$ (d) $\underset{NO}{\underset{|}{CH_2}}-CH_2-\underset{Cl}{\underset{|}{CH_2}}$

52. 1-butyne on oxidation with hot alkaline $KMnO_4$ would yield

(a) $CH_3CH_2CH_2COOH$ (b) CH_3CH_2COOH

(c) $CH_3CH_2COOH + CO_2 + H_2O$ (d) $CH_3CH_2COOH + HCOOH$

53. Which of the following reactions will yield 2, 2-dibromopropane?

(a) $CH_3C \equiv CH + 2HBr \longrightarrow$ (b) $CH_3CH = CHBr + HBr \longrightarrow$

(c) $CH \equiv CH + 2HBr \longrightarrow$ (d) $CH_3CH = CH_2 + HBr \longrightarrow$

54. Which one of the following compounds does not dissolve in conc. H_2SO_4 even on warming?

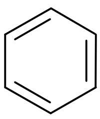
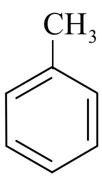
(a) Ethylene (b) Benzene (c) Hexane (d) Aniline

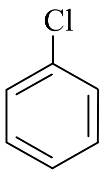
55. The treatment of CH_3MgX with $CH_3C \equiv C-H$ produces

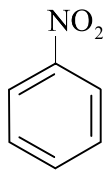
(a) $CH_3-C \equiv CH$ (b) $CH_3-C \equiv C-CH_3$

(c) $CH_3-\underset{H}{\underset{|}{C}}=\underset{H}{\underset{|}{C}}-CH_3$ (d) CH_4

56. Given,

(I) 
(II) 

(III) 

(IV) 

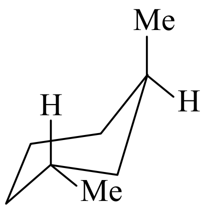
In the above compounds correct order of reactivity in electrophilic substitution reactions will be

(a) II > I > III > IV (b) IV > III > II > I (c) I > II > III > IV (d) II > III > I > IV

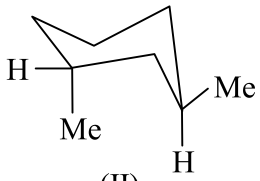
57. 6 L of an alkene require 27 L of oxygen at constant temperature and pressure for complete combustion. The alkene is

(a) ethane (b) propene (c) 1-butene (d) 2-butene

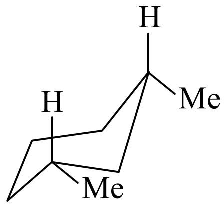
58. Arrange in the correct order of stability (decreasing order) for the following molecules



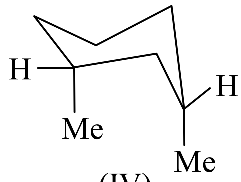
(I)



(II)



(III)

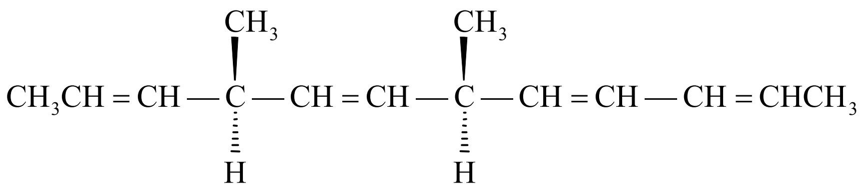


(IV)

(a) (I) > (II) > (III) > (IV) (b) (IV) > (III) > (II) ≈ (I)

(c) (I) > (II) ≈ (III) > (IV) (d) (III) > (I) ≈ (II) > (IV)

59. The number of optically active products obtained from the complete ozonolysis of the given compound is



(a) 0 (b) 1 (c) 2 (d) 4

60. One mole of a symmetrical alkene on ozonolysis gives two moles of an aldehyde having a molecular mass of 44 u. The alkene is

(a) propene (b) 1-butene (c) 2-butene (d) ethane

MATHS

SECTION – III

This section contains ***30 questions.*** Each question has 4 choices (A), (B), (C) and (D) for its answer, out of which ***ONLY ONE*** is correct. Each question carries **+4 marks** for correct answer and **–1 mark** for wrong answer.

61. If $\int_0^x t\,y(t)\,dt = x^2 + y(x)$, then y as a function of x is:

(a) $y = 2-\left(2+a^2\right)e^{\frac{x^2-a^2}{2}}$ (b) $y = 1-\left(2+a^2\right)e^{\frac{x^2-a^2}{2}}$

(c) $y = 2-\left(1+a^2\right)e^{\frac{x^2-a^2}{2}}$ (b) None of the above

62. The population of a country increases at a rate proportional to the number of inhabitants. f is the population which doubles in 30 years, then population will triple in approximately:

(a) 30 years (b) 45 years (c) 48 years (d) 54 years

63. The $x-$ intercept of the tangent to a curve is equal to the ordinate of the point of contact. The equation of the curve through the point (1,1) is:

(a) $ye^{x/y} = e$ (b) $xe^{x/y} = e$ (c) $xe^{y/x} = e$ (d) $ye^{y/x} = e$

64. Consider the two statements:

Statement – I : $y = \sin kt$ satisfies the differential equation $y''+9y=0$.

Statement – II : $y = e^{kt}$ satisfies the differential equation, $y''+y'-6y=0$. The value of $'k'$ for which both the statements are correct is:

(a) −3 (b) 0 (c) 2 (d) 3

65. A function $y = f(x)$ satisfies the differential equation $f(x)\sin 2x - \cos x + \left(1+\sin^2 x\right)f'(x) = 0$ with initial condition $y(0)=0$. The value of $f(\pi/6)$ is equal to:

(a) 1/5 (b) 3/5 (c) 4/5 (d) 2/5

66. For equation of the curve whose subnormal is constant, then:

(a) eccentricity $=1$ (b) eccentricity $=\sqrt{2}$ (c) its axis is x-axis (d) its axis is $y=$axis

67. Let a solution $y=y(x)$ of differential equation, $\sqrt{1-x^2}dy-\sqrt{1-y^2}dx=0$ satisfy $y\left(\frac{1}{2}\right)=1$

Statement-1: $y(x)=\sin\left(\sin^{-1}x+\frac{\pi}{3}\right)$

Statement -2: $y(x)$ is given by $y=\frac{x+\sqrt{3+3x^2}}{2}$

(a) Statement – 1 is true, statement-2 is true and statement-2 is correct explanation for statement – 1

(b) Statement - 1 is true, statement – 2 is true and statement – 2 is NOT the correct explanation for statement – 1.

(c) Statement – 1 is true, statement – 2 is false.

(d) Statement – is False, statement – 2 is true.

68. A normal at $P(x,y)$ on a curve meets the $x-$axis at 'Q' and 'N' is the foot of the ordinate at P. If $NQ=\frac{x(1+y^2)}{(1+x^2)}$, then the equation of curve given that is passes through the point $(3,1)$ is:

(a) $x^2-y^2=8$ (b) $x^2+2y^2=11$ (c) $x^2-5y^2=4$ (d) None of these

69. Let $y=f(x)$ be a continuous function such that $\frac{dy}{dx}=|x-1|$. If $y(0)=0$, then $y(3)$ equals:

(a) $\frac{-3}{2}$ (b) $\frac{3}{2}$ (c) $\frac{5}{2}$ (d) 2

70. A function is continuous and differentiable on R_0 satisfying $xf'(x)+f(x)=1\forall x$ in its domain. If $f(1)=2$, then range of function does not contain:

(a) 1 (b) 2 (c) 3 (d) 4

71. Let 'a' be a positive constant number. Consider two curves $C_1 : y = e^x, C_2 : y = e^{a-x}$. Let S be the area of the part surrounding by C_1, C_2 and y-axis, then $\lim_{a \to 0} \frac{S}{a^2}$ equals:

(a) 4 (b) $\frac{1}{2}$ (c) 0 (d) $\frac{1}{4}$

72. Let S be the area bounded by $y = e^{|\cos 4x|} x = 0, y = 0$ and $x = \pi$. Consider the following four relations

I: $S = 2\int_0^{\pi/2} e^{\cos t} dt$ II: $S = 2\int_0^{\pi/2} e^{\sin t} dt$ III: $S < 2\left(e^{\pi/2} - 1\right)$ IV: $S > \frac{\pi}{2}$

Number of relations which are correct?

(a) 1 (b) 2 (c) 3 (d) 4

73. Area of the region bounded by the curve $= \tan x$ and lines $y = 0$ and $x = 1$ is equal to:

(a) $\int_0^1 \tan(1+x) dx$ (b) $\int_0^{\tan 1} \tan^{-1} y\, dy$ (c) $\int_0^1 \tan^{-1} x\, dx$ (d) $\tan 1 - \int_0^{\tan 1} \tan^{-1} x\, dx$

74. The area bounded by the curve $f(x) = x^3 - 3x$ and $g(x) = 2x^2$ in the second quadrant is:

(a) $\frac{12}{23}$ (b) $\frac{7}{12}$ (c) $\frac{2}{3}$ (d) $\frac{3}{5}$

75. Let A_n be the area of region bounded by a curve $y = x^3\left(1 - x^2\right)^n$ $0 \le x \le 1$ and the $x-$axis, then the value of $\sum_{n=1}^{\infty} A_n$ is equal to:

(a) $\frac{1}{2}$ (b) $\frac{1}{3}$ (c) $\frac{1}{4}$ (d) 1

76. The area of the region(s) enclosed by the curve $y = x^2$ and $y = \sqrt{|x|}$ is:

(a) $\frac{1}{3}$ (b) $\frac{2}{3}$ (c) $\frac{1}{6}$ (d) 1

77. The area bounded by the curves $y = xe^{-x}; xy = 0$ and $x = C$ when C is the x-coordinate of the curve's inflection point, is:

(a) $1-3e^{-2}$ (b) $1-2e^{-2}$ (c) $1-e^{-2}$ (d) 1

78. Find the area lying above the x-axis and under the parabola $y = 4x - x^2$.

(a) $\frac{4}{3}$ (b) $\frac{8}{3}$ (c) $\frac{16}{3}$ (d) $\frac{32}{3}$

79. One of the diagonals of a square is the portion of the line $x/2 + y/3 = 2$ intercepted between the axis. Then the extremities of the other diagonal are

(a) $(5,5),(-1,1)$ (b) $(0,0),(4,6)$ (c) $(0,0),(-1,1)$ (d) $(5,5),(4,6)$

80. If the sum of distances of a point from two perpendicular lines in a plane is 1, then its locus is

(a) a square
(b) a circle
(c) a straight line
(d) two intersecting lines

81. The equation of a line through the point (1, 2) whose distance from the point (3, 1) has the greatest value is

(a) $y = 2x$ (b) $y = x+1$ (c) $x+2y=5$ (d) $y = 3x-1$

82. The centroid of an equilateral triangle is $(0,0)$. If two vertices of the triangle lie on $x + y = 2\sqrt{2}$, then one of them will have its coordinates

(a) $\left(\sqrt{2}+\sqrt{6}, \sqrt{2}-\sqrt{6}\right)$
(b) $\left(\sqrt{2}+\sqrt{3}, \sqrt{2}-\sqrt{3}\right)$
(c) $\left(\sqrt{2}+\sqrt{5}, \sqrt{2}-\sqrt{5}\right)$
(d) none of these

83. If the straight lines $2x+3y-1=0, x+2y-1=0,$ and $ax+by-1=0$ form a triangle with the origin as orthocenter, then (a,b) is given by

(a) $(6,4)$ (b) $(-3,3)$ (c) $(-8,8)$ (d) $(0,7)$

84. The straight lines $4ax+3by+c=0$, where $a+b+c=0$, are concurrent at the point

(a) $(4,3)$ (b) $(1/4, 1/3)$ (c) $(1/2, 1/3)$ (d) none of these

85. The number of values of a for which the lines

$2x+y-1=0,$

$ax+3y-3=0,$

and $3x+2y-2=0$

are concurrent is

(a) 0 (b) 1 (c) 2 (d) infinite

86. The equation of a straight line on which the length of perpendicular from the origin is four units and the line makes an angle of 120° with the x-axis is

(a) $x\sqrt{3}+y+8=0$ (b) $x\sqrt{3}-y=8$ (c) $x\sqrt{3}-y=8$ (d) $x-\sqrt{3}y+8=0$

87. The coordinates of the foot of the perpendicular from the point $(2,3)$ on the line $-y+3x+4=0$ are given by

(a) $(37/10,-1/10)$ (b) $(-1/10,37/10)$ (c) $(10/37,-10)$ (d) $(2/3,-1/3)$

88. The equation to the straight line passing through the point $(a\cos^3\theta, a\sin^3\theta)$ and perpendicular to the line $x\sec\theta+y\,\mathrm{cosec}\,\theta=a$ is

(a) $x\cos\theta-y\sin\theta=a\cos2\theta$ (b) $x\cos\theta+y\sin\theta=a\cos2\theta$

(c) $x\sin\theta+y\cos\theta=a\cos2\theta$ (d) none of these

89. The line $\frac{x}{a}+\frac{y}{b}=1$ meets the $x-$axis at A, the y-axis at B, and the line $y=x$ at C such that area of ΔAOC is twice the area of ΔBOC. Then the coordinates of C are

(a) $\left(\frac{b}{3},\frac{b}{3}\right)$ (b) $\left(\frac{2a}{3},\frac{2a}{3}\right)$ (c) $\left(\frac{2b}{3},\frac{2b}{3}\right)$ (d) none of these

90. A light ray coming along the line $3x+4y=5$ gets reflected from the line $ax+by=1$ and goes along the line $5x-12y=10$. then

(a) $a=64/115, b=112/15$ (b) $a=14/15, b=-8/115$

(c) $a=64/115, b=-8/115$ (d) $a=64/15, b=14/15$

SOLUTION OF AITS JEE(MAIN) PART TEST – 5

PHYSICS

1. **(b)**

There will be no loss of energy if the potential of the spheres is the same i.e. if

$$V = \frac{q}{4\pi\varepsilon_0 r} = \frac{Q}{4\pi\varepsilon_0 R} \text{ or } \frac{q}{r} = \frac{Q}{R}$$

2. **(a)**

The electric potential $V(x, y, z) = 4x^2$,

$$\vec{E} = -\frac{\partial V}{\partial x}\hat{i} = -8x\hat{i} \quad \Rightarrow \quad \vec{E}(1, 2) = -8\hat{i}$$

3. **(c)**

$$E_y = \frac{q_0}{2\pi^2\varepsilon_0 R^2},\ E_z = \frac{q_0}{2\pi^2\varepsilon_0 R^2},\ E_{net} = \sqrt{E_y^2 + E_z^2} = \frac{q_0}{\sqrt{2}\pi^2\varepsilon_0 R^2}$$

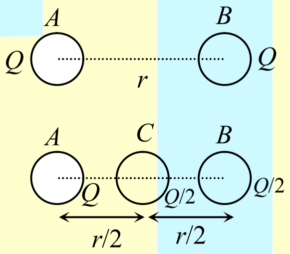
4. **(a)**

Assuming ring as dipole, then dipole moment $\vec{P}$ and $\vec{E}$ are in same direction, so potential energy $U = -PE$

5. **(c)**

$$V_R - V_P = E.\frac{r}{\sqrt{2}}$$

6. (c)



$$F = \frac{Q^2}{4\pi\varepsilon_0 r^2}$$

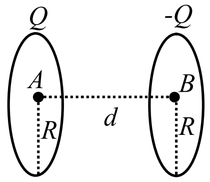
$$F_C = \frac{Q^2/2}{4\pi\varepsilon_0\left(\frac{r}{2}\right)^2} - \frac{Q^2/4}{4\pi\varepsilon_0\left(\frac{r}{2}\right)^2} = \frac{Q^2}{4\pi\varepsilon_0 r^2} = F$$

7. **(a)**

Charge on each drop = q

$$\frac{q_f}{q} = \frac{8q}{q} = 8, \quad 8 \times \frac{4}{3}\pi r^3 = \frac{4}{3}\pi R^3 \Rightarrow R = 2r,\ V = \frac{q}{4\pi\varepsilon_0 r},\ V' = \frac{8q}{4\pi\varepsilon_0 2r},\ \frac{V'}{V} = 4$$

8. **(d)**



$$V_A = \frac{Q}{4\pi\varepsilon_0 R} - \frac{Q}{4\pi\varepsilon_0\sqrt{R^2 + d^2}}$$

$$V_B = -\frac{Q}{4\pi\varepsilon_0 R} + \frac{Q}{4\pi\varepsilon_0\sqrt{R^2+d^2}}$$

$$V_A - V_B = \frac{Q}{2\pi\varepsilon_0}\left(\frac{1}{R} - \frac{1}{\sqrt{R^2+d^2}}\right)$$

9. **(d)**

10. **(a)**

In the given situation potential difference is independent of charge on outer shell.

11. **(a)**

Self energy $= \frac{KQ^2}{2R} + \frac{K(2Q)^2}{2(2R)} = \frac{3}{2}\frac{KQ^2}{R}$

Interaction energy $= \frac{KQ(2Q)}{2R} = \frac{KQ^2}{R}$

Total $= \frac{5}{2}\frac{KQ^2}{R}$

12. **(c)**

$$\frac{K(4e)q}{(x-y)^2} = \frac{Kqe}{y^2}, \quad y = \frac{x}{3}$$

13. **(d)**

On closing the switch potential of outer shell is zero

$\frac{KQ}{2R} + \frac{KQ_1}{2R} = 0, = -Q$ charge flow $= 3Q$

14. (b)

Potential energy of the system when charge Q is at O is $U_0 = \frac{qQ}{a} + \frac{qQ}{a} = \frac{2qQ}{a}$

When charge Q is shifted to position O', the potential energy will be

$$U = \frac{qQ}{(a+x)} + \frac{qQ}{(a-x)} = \frac{qQ(2a)}{(a^2-x^2)} = \frac{2qQ}{a} \times \left(1 - \frac{x^2}{a^2}\right)^{-1}$$

$$= \frac{2qQ}{a} \times \left(1 + \frac{x^2}{a^2}\right) \qquad (\because x << a)$$

$\therefore\ \Delta U = U - U_0 = \frac{2qQ}{a}\left(1 + \frac{x^2}{a^2}\right) - \frac{2qQ}{a} = \frac{2qQ}{a^3}(x^2)$ Hence ΔU $\Delta U \propto x^2$.

15. **(b)**

$$V_1 = \frac{1}{4\pi\varepsilon_0}\left(\frac{Q_1}{R} + \frac{Q_2}{\sqrt{2}\,R}\right), \qquad V_2 = \frac{1}{4\pi\varepsilon_0}\left(\frac{Q_2}{R} + \frac{Q_1}{\sqrt{2}\,R}\right)$$

$$\therefore \text{ Work done} = q\,(V_1 - V_2) = \frac{q}{4\pi\varepsilon_0}\left[\left(\frac{Q_1}{R} + \frac{Q_2}{\sqrt{2}\,R}\right) - \left(\frac{Q_2}{R} + \frac{Q_1}{\sqrt{2}\,R}\right)\right]$$

$$= \frac{q}{4\pi\varepsilon_0\sqrt{2}R}\,(Q_1 - Q_2)\,(\sqrt{2} - 1)$$

16. **(c)**

$$E \text{ at axis of ring } = \frac{Kqx}{\left(R^2 + x^2\right)^{\frac{3}{2}}}$$

For $E_{\max}$, $\dfrac{dE}{dx} = 0$, $\qquad x = \pm\dfrac{R}{\sqrt{2}}$

$$E_{\text{maximum}} = \frac{1}{4\pi\varepsilon_0}\frac{qR/\sqrt{2}}{\left(R^2 + \frac{R^2}{2}\right)^{\frac{3}{2}}} = \frac{1}{4\pi\varepsilon_0}\frac{2qR}{3\sqrt{3}R^2}$$

17. **(d)**

$$W_{\text{electric field}} = \int q\vec{E}.\left(dx\hat{i} + dy\hat{j} + dz\hat{k}\right) = \int_{(2,2)}^{(4,1)} y\,dx + x\,dy = \int_{(2,2)}^{(4,1)} d(xy) = xy\Big|_{(2,2)}^{(4,1)} = 0$$

18. **(d)**

$$\frac{1}{2}mv^2 = \frac{KqQ}{r}$$

$$\frac{1}{2}m(2v)^2 = \frac{KqQ}{r_1} \Rightarrow r^1 = \frac{r}{4}$$

19. **(a)**

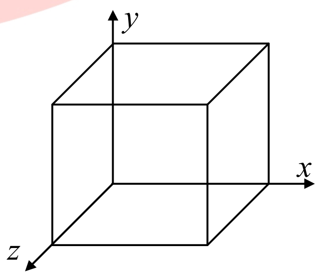
$$\frac{q^2}{4\pi\varepsilon_0 a} = k$$

20. **(b)**

Net flux in x-direction = 0, Net flux in y-direction

$= A[3(1)^2 + 2] - A[3(0) + 2]$

$\Rightarrow \dfrac{q}{\varepsilon_0} = 3A \qquad \Rightarrow \qquad q = 3\in_0 A = 3\in_0$, (as $A = 1m^2$)



21. **(c)**

$\tan 30° = \frac{1}{2}\tan\theta$, $\tan\theta = \frac{2}{\sqrt{3}}$

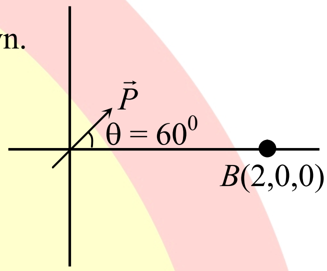
22. **(d)**

23. **(b)**

$$\vec{E} = -\left[\frac{\partial V}{\partial x}\hat{i} + \frac{\partial V}{\partial y}\hat{j}\right] = K\left(y\hat{i} + x\hat{j}\right) \therefore E = \sqrt{E_x^2 + E_y^2} = \sqrt{(Ky)^2 + (Kx)^2} = Kr \quad \text{i.e., } E \propto r$$

24. **(c)**

The dipole moment makes an angle 60^0 with x-axis and lies in x-y plane as shown.



The electric field a point B due to dipole is

$$E = \frac{kP}{r^3}\sqrt{1 + 3\cos^2\theta} \text{ where } \theta = 60^0$$

$$\therefore \; E = \frac{\sqrt{7}k}{8}$$

25. **(b)**

By Guass theorem, $\oint EdS = \frac{1}{\varepsilon_0}\int dq$

$$\Rightarrow E \times 4\pi r^2 = \frac{4\pi\rho_0}{\varepsilon_0}\int_0^r\left(1 - \frac{x}{3}\right)x^2 dx \qquad \Rightarrow E = \frac{\rho_0}{\varepsilon_0}\left[\frac{r}{3} - \frac{r^2}{12}\right]$$

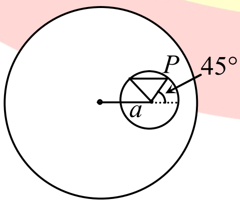
for max. electric field, $\frac{dE}{dr} = 0$

$$\Rightarrow \frac{1}{3} - \frac{r}{6} = 0$$

$$\Rightarrow r = \frac{6}{3} = 2$$

26. **(a)**

$$F = eE = \frac{e\rho a}{3\varepsilon_0}$$



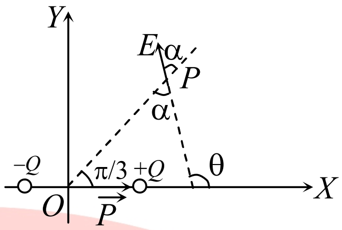
$$r\sqrt{2} = \frac{1}{2}\frac{F}{m}t^2$$

$$t = \sqrt{\frac{6\sqrt{2}r\varepsilon_0 m}{e\rho a}}$$

27. **(b)**

$$\theta = \frac{\pi}{3} + \alpha \text{ where } \tan\alpha = \frac{1}{2}\tan\theta = \frac{1}{2}\tan\frac{\pi}{3}$$

$$\Rightarrow \alpha = \tan^{-1}\frac{\sqrt{3}}{2} \quad \text{so, } \theta = \frac{\pi}{3} + \tan^{-1}\frac{\sqrt{3}}{2}$$



28. **(b)**

$$F = \frac{\Delta P}{\Delta t},\ F = (q_1 + q_2)\,E_0,\ \ \Delta t = 2t_0,$$

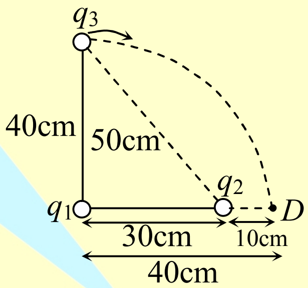
$$(q_1+q_2)\,E_0 = \frac{\Delta P}{2t_0}, \qquad \Delta P = 2\,(q_1 + q_2)\,E_0\,t_0$$

29. **(a)**

Change in potential energy $(\Delta U) = U_f - U_i$

$$\Rightarrow \Delta U = \frac{1}{4\pi\varepsilon_0}\left[\left(\frac{q_1q_3}{0.4} + \frac{q_2q_3}{0.1}\right) - \left(\frac{q_1q_3}{0.4} + \frac{q_2q_3}{0.5}\right)\right]$$

$$\Rightarrow \Delta U = \frac{1}{4\pi\varepsilon_0}[8q_2q_3] = \frac{q_3}{4\pi\varepsilon_0}(8q_2) \therefore\ k = 8q_2$$



30. **(c)**

Potential of centre of sphere $= \frac{Kq}{r} + V_i = \frac{Kq}{r}$

where V_i = potential due to induced charge at centre = 0 [$\therefore\ \Sigma q_i = 0$ and all induced charges are equidistance from centre]

$\therefore$ potential at point $P = \frac{Kq}{r} = \frac{Kq}{r_1} + V_i$ (For conductor all points are equipotential)

$$\therefore\ V_i = K\left(\frac{q}{r} - \frac{q}{r_1}\right)$$

CHEMISTRY

31. **(c)** 32. **(d)** 33. **(d)** 34. **(c)**

35. **(b)** 36. **(d)**

37. **(b)**

$NH_4^+ + OH^- \xrightarrow{k_1} NH_4OH \qquad k_1 = 4 \times 10^{10}$

$NH_4OH \underset{K_b}{\overset{K_f}{\rightleftharpoons}} NH_4^+ + OH^-$

$$k_{eq} = \frac{k_f}{k_b}$$

$$1.8 \times 10^{-5} = \frac{k_f}{4 \times 10^{10}} \Rightarrow k_f = 7.2 \times 10^5$$

38. **(a)** 39. **(b)** 40. **(a)**

41. **(c)**

Only red P and conc. HI convert –COOH group to $-CH_3$.

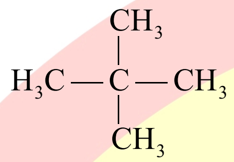
42. **(c)**

$$CH_3COONa + NaOH \xrightarrow[\Delta]{\text{Sodalime}} CH_4 + Na_2CO_3$$

43. **(d)**

As the number of carbon atom increases, boiling point increases. Boiling point decreases with branching

2, 2 dimethyl propane



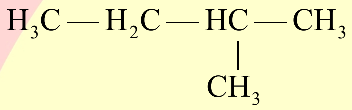
Boiling point = 282.5 K

n-pentane

$$H_3C-CH_2-CH_2-CH_2-CH_3$$

Boiling point = 309.1 K

2-methyl butane



Boiling point = 301 K

n-butane

$$H_3C-H_2C-CH_2-CH_3$$

Boiling point = 273 K

(4 carbon atoms with no branching)

44. **(b)**

The cetane number is defined as the percentage by volume of cetane in a mixture of cetane (n-hexadecane) and α-methyl naphthalene.

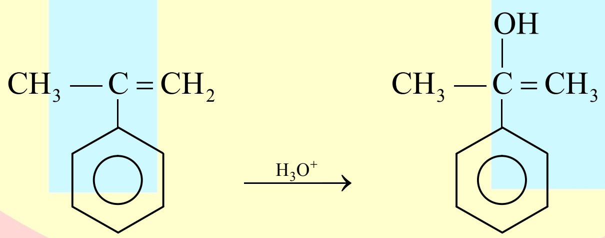
Since, in the given mixture, percentage of α-methyl naphthalene = 10%

$\therefore$ The percentage of cetane = (100 – 10)% = 90%

$\therefore$ The cetane number = 90

45. **(a)**

Acidic hydration of 2-phenylpropene follows electrophilic reaction mechanism forming an intermediate 3° carbocation (more stable), thereby forming 2-phenyl-2-propanol.



46. **(b)**

$$\underset{\text{n-propyl bromide}}{CH_3CH_2CH_2Br} \xrightarrow{\text{Alc.KOH}} \underset{\text{Propene}}{CH_3CH = CH_2}$$

47. **(b)**

$$CH_2 = CH_2 \xrightarrow[273K]{KMnO_4} \underset{\text{Ethylene glycol}}{\underset{OH\quad OH}{CH_2 - CH_2}}$$

$CH_2 = CH_2 \xrightarrow{AgNO_3}$ No reaction

48. **(c)**

$$\underset{\text{2, 3-dimethyl-1-butene}}{CH_3-\underset{\underset{CH_3}{|}}{CH}-\underset{\underset{CH_3}{|}}{C}=CH_2} \xrightarrow{(i)O_3;\,(ii)\,Zn/H_2O} \underset{\text{3-methyl-2-butanone}}{CH_3-\underset{\underset{CH_3}{|}}{CH}-\underset{\underset{CH_3}{|}}{C}=O} + \underset{\text{Methanal}}{H-\underset{\underset{H}{|}}{C}=O}$$

49. **(d)**

$$\underset{\text{Propene}}{CH_3-CH=CH_2} \xrightarrow[\text{Markownikoff's rule}]{\overset{\delta-}{HO}-\overset{\delta+}{Br}} \underset{\text{1-bromo-2-propanol}}{CH_3\underset{\underset{OH}{|}}{CH}-\underset{\underset{Br}{|}}{CH_2}}$$

50. **(a)** 51. **(a)** 52. **(c)** 53. **(a)**
54. **(c)** 55. **(d)** 56. **(a)** 57. **(b)**
58. **(d)** 59. **(a)** 60. **(c)**

MATHS

61. **(a)**

Differentiating both sides $\Rightarrow xy(x) = 2x + y'(x)$

Hence, $\frac{dy}{dx} - xy = -2x$, where $y'(x) = \frac{dy}{dx}$ and $y(x) = y$

I. F. $= e^{\int -x\,dx} = e^{-x^2/2}$

So, $ye^{-x^2/2} \int -2xe^{-x^2/2}dx$

$ye^{-x^2/2} = 2e^{-x^2/2} + C$

If $x = a, a^2 + y = 0 \Rightarrow y = -a^2$

Hence, $-a^2 = 2 + ce^{\frac{a^2}{2}} \Rightarrow c = -(2+a^2)e^{-a^2/2}$

$y = 2 - (2+a^2)e^{\frac{x^2-a^2}{2}}$

62. **(c)**

Let population $= x$ at time 't' years

Given, $\frac{dx}{dt} \propto x$

$\Rightarrow \frac{dx}{dt} = kx$

$\Rightarrow \int \frac{dx}{x} = \int k\,dt \Rightarrow \ln x = kt + \ln c$

$\Rightarrow x = ce^{kt}$

If $t=0, x=x_0; c=x_0; x=x_0e^{kt}$, given $x=2x_0$ at $t=30$

$2x_0 = x_0e^{30k} \Rightarrow 2=e^{30k}$

$\therefore \ln 2 = 30k$

$x=3x_0$ at t .... (1)

$3x_0 = x_0e^{kt} \Rightarrow 3 = e^{kt}$

$\therefore kt = \ln 3$

Dividing equation (ii) by (I); then $\frac{t}{30} = \frac{\ln 3}{\ln 2}$ $or\, t = 48$ years

63. **(a)**

$Y - y = \frac{dy}{dx}(X - x)$ for x-intercept, $y=0$

$\Rightarrow X = x - y\frac{dx}{dy}$

Given: $x - y\frac{dx}{dy} = y$

$\Rightarrow \frac{dy}{dx} = \frac{y}{(x-y)}$

Putting $y = vx$, we get

$\Rightarrow v + x\frac{dv}{dx} = \frac{v}{(1-v)}$

$\Rightarrow x\frac{dv}{dx} = \frac{v^2}{(1-v)}$

$\Rightarrow \int\left(\frac{1-v}{v^2}\right)dv = \int\frac{dx}{x}$

$\Rightarrow -\frac{1}{v} - \ln v = \log x + c$

$-\frac{1}{v} = \log(vx) + c$

$\Rightarrow -\frac{x}{y} = \log y + c$

Given $x=1, y=1$ so, $c=-1$, equation of curve is $1-\frac{x}{y} = \log y$

$\Rightarrow ye^{x/y} = e$

64. **(a)**

Statement-1: $y=\sin kt,\ y''=-k^2\sin\ kt$

$\therefore\ \ y''+9y=0\Rightarrow -k^2\sin kt+9\sin kt=0 \qquad \Rightarrow \qquad k=0,3,-3$

Statement -2: $y=e^{kt}, y'=ke^{kt}, y'=k^2e^{kt}$

$\therefore\ \left(k^2+k-6\right)e^{kt}=0$

$\Rightarrow \qquad (k+3)(k-2)e^{kt}=0$

$\Rightarrow \qquad k=-3$ or 2 $\qquad$ Common value is $k=-3$.

65. (d)

Let $y=f(x)\dfrac{dy}{dx}=f'(x);$

$y\sin 2x-\cos x+\left(1+\sin^2 x\right)\dfrac{dy}{dx}=0$

$\left(1+\sin^2 x\right)\dfrac{dy}{dx}+\sin 2xy=\cos x$

$\dfrac{dy}{dx}+\dfrac{\sin 2x}{\left(1+\sin^2 x\right)}y=\dfrac{\cos x}{\left(1+\sin^2 x\right)}$

If $=e=e\int\dfrac{\sin 2x}{1+\sin^2 x}=e\ln\left(1+\sin^2 x\right)$

$=1+\sin^2 x.$

So, $y\left(1+\sin^2 x\right)=\int\cos x\,dx=\sin x+c$

As $x=0, y=0, c=0\ \Rightarrow\ y\left(1+\sin^2 x\right)=\sin x$

At $x=\dfrac{\pi}{6}\Rightarrow y\left(\dfrac{5}{4}\right)=\dfrac{1}{2}\Rightarrow y=\dfrac{2}{5}$

66. **(b)**

We have been given that subnormal is constant. We known for any curve,

Length of subnormal $=y\dfrac{dy}{dx}=k$ (constant)

So, variable separable $y\,dy=k\,dx$, now integrating

$\int x\,dy=\int k\,dx$

$\Rightarrow \dfrac{y^2}{2}=kx+C$

$\Rightarrow \quad y^2 = 2kx + 2C$, comparing it with

$y^2 = 2ax + b$

So, $a = k$ and $b = 2C$.

Since it is a parabola so eccentricity will be unity.

67. **(c)**

$$\sqrt{(1-x^2)}\ dy - \sqrt{(1-y^2)}\ dx = 0$$

$$\int \frac{dy}{(1-y^2)} = \int \frac{dx}{\sqrt{(1-x^2)}}$$

$$\sin^{-1}(y) = \sin^{-1} x + C; \qquad \text{at } x = \frac{1}{2}; y = 1$$

$$\frac{\pi}{2} = \frac{\pi}{6} + C;\ C = \frac{\pi}{3}$$

$$\sin^{-1}(y) = \sin^{-1} x + \frac{\pi}{3}$$

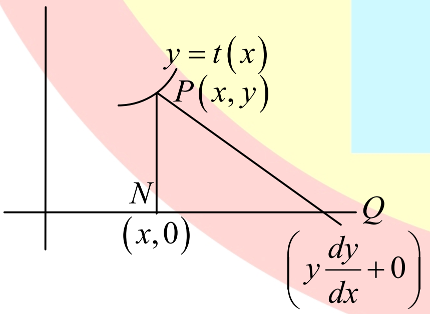
Thus, $\quad y = \sin^{-1}\left(x + \frac{\pi}{3}\right)$

$$y = \sin(\sin^{-1} x)\cos\frac{\pi}{3} + \cos(\sin^{-1} x)\sin\frac{\pi}{3}$$

$$y = x\frac{1}{2} + \sqrt{(1-x^2)}\frac{\sqrt{3}}{2}$$

68. **(c)**

Equation of normal $Y - y = -\frac{dx}{dy}(X - x)$



$$NQ = y\frac{dy}{dx} = \frac{x(1+y^2)}{(1+x^2)} \Rightarrow \int \frac{x\,dx}{(1+x^2)} = \int \frac{y\,dy}{(1+y^2)}$$

$$\Rightarrow \ln(1+x^2) = \ln(1+y^2) + \ln c \Rightarrow (1+y^2) = \frac{(1+x^2)}{c}$$

As curve passes through (3, 1)

$$1+(1)^2=\frac{1+(3)^2}{c}=\frac{1+9}{c}=\frac{10}{c}$$

$$2=\frac{10}{c}\Rightarrow c=5 \Rightarrow 1+y^2=\frac{1+x^2}{5}, 5+5y^2=1+x^2 \Rightarrow x^2-5y^2=4$$

69. **(c)**

$$\frac{dy}{dx}=|x-1|, y(0)=0$$

$$\frac{dy}{dx}=\begin{cases}(x-1) & ; \ x\ge 1\\ -(x-1) & ; \ x<1\end{cases}$$

$$y=\begin{cases}\frac{x^2}{2}-x+C_1 & ; \ x\ge 1\\ -\frac{x^2}{2}+x+C_2 & ; \ x<1\end{cases}$$

$$y(0)=0; C_2=0$$

$$y=\begin{cases}\frac{x^2}{2}-x+C_1 & ; \ x\ge 1\\ -\frac{x^2}{2}+x & ; \ x<1\end{cases}$$

Function should be continuous at $x=1$.

$$\frac{1}{2}-1+C_1=-\frac{1}{2}+1; \qquad C_1=-1+2=1$$

$$y=\begin{cases}\frac{x^2}{2}-x+1 & ; \ x\ge 1\\ -\frac{x^2}{2}+x & ; \ x<1\end{cases}$$

$$y(3)=\frac{9}{2}-3+1=\frac{5}{2}$$

70. **(a)**

$$xf'(x)+f(x)=1$$

$$x\frac{dy}{dx}+y=1$$

$$x\,dy+y\,dx=dx$$

$$\int d(xy)=\int dx$$

$xy = x + C$

At $x = 1; y = 2; C = 1$

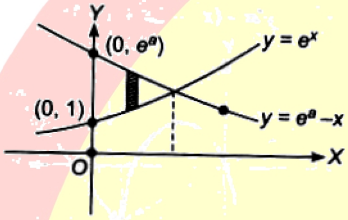
$xy = (x+1)$

$y = \frac{(x+1)}{x}$; $x = \frac{1}{y-1}$; $y \neq 1$

Range is $R - \{1\}$.

71. **(d)**

Solving $e^x = e^{a-x}$, we get



$e^{2x} = e^a \Rightarrow x = \frac{a}{2}$

$S = \int_0^{a/2} \left(e^a e^{-x} - e^x\right) dx = -\left[e^a e^{-x} + e^x\right]_0^{a/2}$

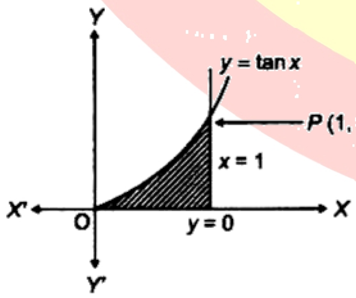
$S = \left(e^a + 1\right) - \left(e^{a/2} + e^{a/2}\right) = e^a - 2e^{a/2} + 1 = \left(e^{a/2} - 1\right)^2$

$\therefore \frac{S}{a^2} = \left(\frac{e^{a/2} - 1}{a}\right)^2 = \frac{1}{4}\left(\frac{e^{a/2} - 1}{a/2}\right)^2$

$\lim_{a \to 0} \frac{S}{a^2} = \frac{1}{4}$

72. **(d)**

73. **(a)**



$A = \int_0^1 \tan x \, dx$

Applying King's property

$$A=\int_0^1 \tan(1-x)\,dx$$

$$x=\tan^{-1} y$$

$$A=\int_0^{\tan 1}\left(1-\tan^{-1} y\right)dy=\tan 1-\int_0^{\tan 1}\tan^{-1} y\,dy$$

74. **(b)**

75. **(c)**

$$y=x^3\left(1-x^2\right)^n, 0\le x\le 1$$

$$\sum_{n=1}^{\infty} A_n \int_0^1 x^3\left(1-x^2\right)^1 dx+\int_0^1 x^3\left(1-x^2\right)^2 dx+\ldots\infty$$

$$=\int_0^1 \frac{x^3\left(1-x^2\right)}{1-\left(1-x^2\right)}dx=\int_0^1 x\left(1-x^2\right)dx=\left[\frac{x^2}{2}-\frac{x^4}{4}\right]_0^1$$

$$=\frac{1}{2}-\frac{1}{4}=\frac{1}{4}$$

76. **(b)**

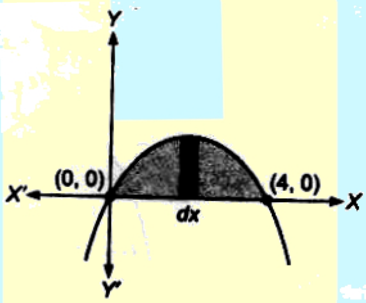
77. **(a)**

78. **(d)**

Required area $=\int_0^4 y\,dx$

$$=\int_0^4\left(4x-x^2\right)dx=\left[2x^2-\frac{x^3}{3}\right]_0^4$$

$$32-\frac{64}{3}=32\left(\frac{3-2}{3}\right)=\frac{32}{3}$$



79. **(a)**

The extremities of the given diagonal are (4, 0) and (0, 6). Hence, the slope of this diagonal is $-3/2$ and the slope of other diagonal is 2/3. The equation of the other diagonal is

$$\frac{x-2}{3/\sqrt{13}}=\frac{y-3}{2/\sqrt{13}}=r$$

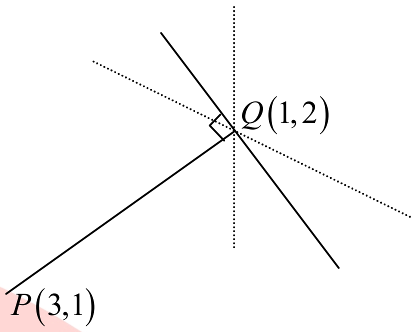
For the extremities of the diagonal, $r=\pm\sqrt{13}$. Hence, $x-2=\pm 3, y-3=\pm 2$

$x=5,-1$ and $y=5,1$

Therefore, the exteremities of the diagonal are (5, 5) and $(-1,1)$.

80. **(a)**

81. **(a)**



Obviously, the line through Q is at the greater distance from point P when it is perpendicular to PQ. Now, the slope of line PQ is $m_{PQ} = -1/2$. Then the slope of perpendicular line passing through Q is

$y-2=2(x-1)$ or $2x-y=0$

82. **(a)**

83. **(c)**

84. **(b)**

The set of lines is $4ax+3by+c=0$, where $a+b+c=0$. Eliminating c, we get

$4ax+3by-(a+b)=0$

or $a(4x-1)+b(3y-1)=0$

This passes through the intersection of the lines $4x-1=1=0$ and $3y-1=0$, i.e., $x=1/4, y=1/3$, i.e., $(1/4,1/3)$.

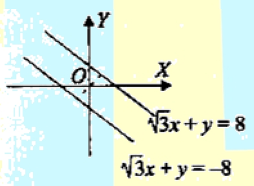
85. **(d)**

All values of a.

86. **(a)**

Given that the slope is $-\sqrt{3}$. Therefore, the line is

$y=-\sqrt{3}\,x+c$



or $\sqrt{3}x+y=c$

Now, $\left|\frac{c}{2}\right|=4$ or $c=\pm 8$

Or $x\sqrt{3}+y=\pm 8$

87. **(b)**

The line passing through (2, 3) and perpendicular to $-y+3x+4=0$ is $\frac{y-3}{x-2}=-\frac{1}{3}$

or $3y+x-11=0$

Therefore, the foot is $x=-1/10, y=37/10$.

88. **(a)**

The line perpendicular to $x\sec\theta+y\operatorname{cosec}\theta=a$ is $x\operatorname{cosec}\theta-y\sec\theta=\lambda$

This line passes through the point $(a\cos^3\theta, a\sin^3\theta)$ Then, $(a\cos^2\theta)\operatorname{cosec}\theta-(a\sin^3\theta)\sec\theta=\lambda$

Or $\lambda = a\left(\frac{\cos^3\theta}{\sin\theta} - \frac{\sin^3\theta}{\cos\theta}\right) = a\frac{\cos 2\theta}{\cos\theta\sin\theta}$

89. **(c)**

90. **(c)**

$ax + by = 1$ will be one of the bisectors of the given line. Equations of bisectors of the lines are

$$\frac{3x+4y-5}{5} = \pm\left(\frac{5x-12y-10}{13}\right)$$

or $64x - 8y = 115$

or $14x + 112y = 15$

or $a = \frac{64}{115}, b = -\frac{8}{115}$

or $a = \frac{14}{15}, b = \frac{12}{115}$